Solve the Second Order Ordinary Differential Equations by Adomian Decomposition Method

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DOI: https://doi.org/10.5281/zenodo.7907545

Published Date: 08-May-2023

Abstract: **The Adomian decomposition model (ADM) is described by Evans and Raslan (2005) as a semi-analytical model used to solve partial and ordinary non-linear differentials. ADM was designed by George Adomian between the 1970s and the 1990s (Hosseini & Nasabzadeh, 2007) whilst working at the University of Georgia's Applied Mathematics department.**

Keywords: **Ordinary Differential - Differential Equations - Adomian Decomposition Method.**

1. INTRODUCTION

The ADM model works under the fundamental idea of decomposing the unknown function u (x, y) of any equation into a sum of infinite components informed by the decomposition series. It has been established that the ADM has proved to be an effective approach when solving broad classes of non-linear partial and ordinary differential (Evans & Raslan, 2005). It is based on decomposing the solution into an infinite series of terms that can be computed recursively. As such, the idea behind ADM is to decompose the non-linear differential equation into a series of simpler linear equations, which can then be solved using standard techniques.

2. SOLVING FOR THE SECOND-ORDER ODES BY ADM

To solve a second-order ordinary differential equation using ADM, we can follow these steps:

Step 1: Write the DE in the form:

$y'' = f(y, y', x)$

This step requires one to express the second-order differential equation in terms of the second derivative of the dependent variable y, y', and x. For example, consider the following second-order differential equation:

$y'' + 4y' + 4y = 4x^2$

This equation can thus be rewritten to take the form:

$y'' = f(y, y', x)$, by moving the other terms to the right-hand side:

$y'' = 4x^2 - 4y' - 4y$

Step 2: Apply the ADM to the equation to obtain a series solution

This is the stage where the ADM is applied to the equation. The ADM represents the solution of a non-linear differential equation as an infinite series of Adomian polynomials. The general form of the solution is:

y(x) = Σn=0∞ un(x)

Here, un(x) represents the nth-order Adomian polynomial. The Adomian polynomials are defined recursively as follows:

u0(x) = G(x)

un(x) = -(1/n) * L(f(u0, u1, ..., un-1))

Here, G(x) represents the initial condition, and L is a differential operator that operates on the function f. In the ADM, the differential operator L is typically chosen as the inverse of the linear operator in the differential equation.

Step 3: Calculate the Adomian polynomials recursively

Given the understanding of the Adomian polynomials, we can calculate the Adomian polynomials recursively. Starting with u0(x) = G(x), we can obtain the subsequent Adomian polynomials using the formula:

un(x) = -(1/n) * L(f(u0, u1, ..., un-1))

Where L is the inverse operator of the linear operator in the differential equation. It is thus possible to calculate the Adomian polynomials up to the desired order.

Step 4: Substitute the Adomian series solution into the original DE and equate the coefficients of each power of x to 0

Upon obtaining the Adomian series solution, we can substitute it into the original differential equation and equate the coefficients of each power of x to zero. This will generate a set of linear algebraic equations for the coefficients of the Adomian polynomials.

Step 5: Solve the resulting linear equations for the coefficients of the Adomian polynomials

The linear equations obtained in the previous step can be solved for the coefficients of the Adomian polynomials. Once the coefficients are obtained, the Adomian series solution can be computed.

Step 6: Substitute the coefficients into the Adomian series solution to obtain the final solution

In this step, we substitute the coefficients obtained in the previous step into the Adomian series solution. This gives us the final solution to the differential equation.

3. EXAMPLE

Given an ordinary differential equation:

y” - y’ - 2y = sin(x)

Step 1: Write the differential equation in the form y” = f(y, y’, x)

We can rewrite the differential equation as:

y” = sin(x) + y’ + 2y

Step 2: Apply the Adomian decomposition method to the equation to obtain a series solution

We assume that the solution to the differential equation can be written in the form:

y(x) = Σn=0∞ un(x)

where un(x) is the nth-order Adomian polynomial. We can calculate the Adomian polynomials recursively using the following formula:

u0(x) = G(x) un(x) = -(1/n) * L(f(u0, u1, ..., un-1))

where G(x) is the initial condition and L is the inverse of the linear operator in the differential equation.

International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 11, Issue 1, pp: (6-9), Month: April 2023 - September 2023, Available at: www.researchpublish.com

Step 3: Calculate the Adomian polynomials recursively

Using the formula for the Adomian polynomials, we can calculate the Adomian polynomials recursively. Starting with u0(x) = G(x), we can obtain the subsequent Adomian polynomials as follows:

u0(x) = 0 u1(x)

= sin(x) u2(x)

= -(1/2)cos(x) u3(x)

= (1/2)sin(x) - (1/3)cos(x) u4(x)

= (1/8)cos(x) + (5/24)sin(x) + (1/12)cos(x)

We can continue calculating the Adomian polynomials up to the desired order.

Step 4: Substitute the Adomian series solution into the original DE and equate the coefficients of each power of x to zero

Next, we substitute the Adomian series solution into the original DE, equating each power of x coefficients to zero. This generates a set of linear algebraic equations for the coefficients of the Adomian polynomials. For example, equating the coefficients of x^0 gives:

u0" = 0 u1" - u1' - 2u0

= sin(x) u2" - u2' - 2u1

= 0 u3" - u3' - 2u2

= 0 u4" - u4' - 2u3

= 0 ...

Equating the coefficients of x^1 gives:

u0' = 0 u1' - u0

= 0 u2' - u1

= 0 u3' - u2

= 0 u4' - u3

= 0 ...

Step 5: Solve the resulting linear equations for the coefficients of the Adomian polynomials

Solving the linear equations obtained in the previous step, we obtain the coefficients of the Adomian polynomials. For example, solving the equation for u1" - u1' - 2u0 = sin(x) gives:

u1(x) = sin(x)

Using this value, we can solve for the other coefficients and obtain the Adomian series solution.

Step 6: Substitute the coefficients into the Adomian series solution to obtain the final solution

Finally, we substitute the coefficients obtained in the previous step into the Adomian series solution to obtain the final solution. For this example, the final solution is:

y(x) = u0(x) + u1

4. CRITICAL ANALYSIS

The decomposition of the non-linear second-order ordinary differentials using the Adomian decomposition method shows that the model is crucial in the decomposition of these differentials. However, the ADM has several strengths and weaknesses as a method of solving second-order ordinary differential equations.

The ADM has several strengths that must be considered (Biazar, Babolian, & Islam, 2004). The ADM model is indispensable when it comes to solving non-linear problems. The model can be applied to a wide range of non-linear problems, including those that are difficult to solve using traditional methods such as the Runge-Kutta method or the finite element method (Fadugba, Zelibe, & Edogbanya, 2013). Studies have established that the ADM model has a high degree of accuracy in many cases, and it can provide solutions that are very close to the exact solution of the differential equation. Hosseini and Nasabzadeh (2006) also assert that the model does not need linearisation. ADM does not require the problem to be linearised, which is a major advantage over other methods that rely on linearisation techniques. Lastly, the ADM can be extended to higher-order differential equations, partial differential equations, and other types of equations, making it a versatile method for solving a wide range of mathematical problems (Fadugba, Zelibe, & Edogbanya, 2013).

Despite these inherent strengths, the method could be developed further, or enhanced to overcome some of the prevalent weaknesses it bears. For example, ADM may not converge for some problems, especially if the problem has singularities or if the Adomian polynomials are difficult to calculate (Hosseini & Nasabzadeh, 2006). In such cases, the method may require modifications or may not be suitable at all. It is also a challenge to determine the initial conditions for the ADM to be absorbed. Difficulty in determining the initial conditions: ADM may require additional information to determine the initial conditions, especially if the problem is non-linear or if the initial conditions are not well defined (Biazar, Babolian, & Islam, 2004). Lastly, the method has been accused of being overly dependent on the choice of Adomian polynomials: The choice of Adomian polynomials can affect the accuracy and convergence of the method. In some cases, it may be difficult to find a suitable set of Adomian polynomials, which can limit the applicability of the method.

5. CONCLUSIONS

This report finds that the ADSM is a powerful technique for solving non-linear differential equations. The method involves decomposing the differential equation into a series of simpler linear equations, which can then be solved using standard techniques. The Adomian decomposition method provides a general and efficient approach to solving non-linear differential equations that do not have analytical solutions.

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